Random Graphs
Exercise Sheet 4

Question 1. Show if every two disjoint vertex sets of size m contain an edge between them then G contains a path of length $n - 2m$.

Show that for every $\varepsilon > 0$ there is a $c > 0$ such that if $p = \frac{c}{n}$ then with high probability $G_{n,p}$ contains a path of length $(1 - \varepsilon)n$.

Question 2. As in Lemma 6.2 show that if $p = \frac{1}{n}(\log n - \log\log n)$ then with high probability $G_{n,p}$ contains no connected component of size between 2 and $\frac{n}{2}$.

Question 3. The *k-core* of a graph G is the maximal induced subgraph of G with minimum degree at least k.

Show that if c is large enough then with high probability $G_{n,\frac{c}{n}}$ has a non-empty k-core. Show further that it is linear in size.

Question 4. Let $p = \frac{c}{n}$ with $c > 1$. Recall that with high probability there is a unique 'giant' component of $G_{n,p}$ of size $(1 + o(1))\beta_c n$ for some β_c. How many edges are in the giant component?

(It may be easier to consider the $G_{n,m}$ model)

Question 5. Show that for sufficiently large C and $p = \frac{C^2}{n}$ with high probability every 2-colouring of the edges of $G_{n,p}$ contains a monochromatic path of length at least $\frac{n}{C}$.